Lecture #1: How High Did the Rocket Go?

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New England Sci - Tech

Lecture Series in Elementary Mathematics in Modeling Rocket Flight

July 21st, 2022

Altitude Calculations

- Measure to find how far from the launcher you are going to stand when the rocket is launched. In other words, you will be measuring the BASELINE.

Trigonometry

$\boxed{\begin{array}{l} \sin\theta \\ \cos\theta \end{array}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

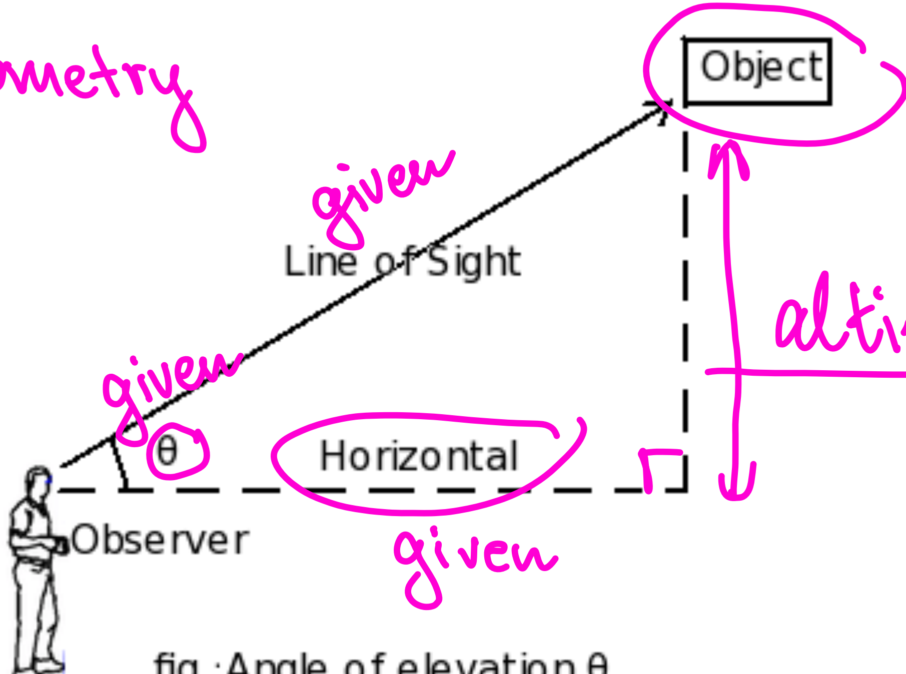
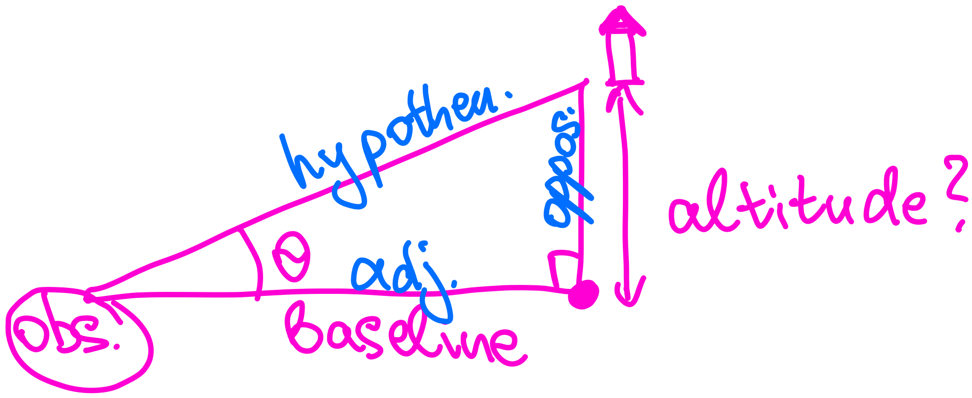


fig.:Angle of elevation θ



$$\tan\theta = \frac{opp}{adj} = \frac{altitude}{baseline}$$

$$\frac{\tan\theta}{1} = \frac{altitude}{baseline}$$

$$altitude \cdot 1 = \tan\theta \cdot baseline$$

$$\boxed{altitude = \tan\theta \cdot baseline}$$

- The second measure you need is the ANGULAR DISTANCE the rocket travels from launch to apogee (**the highest point of flight**).

  The angular distance is an angle of elevation θ.
- Now, to calculate the **altitude** we need to use some basic TRIGONOMETRY.

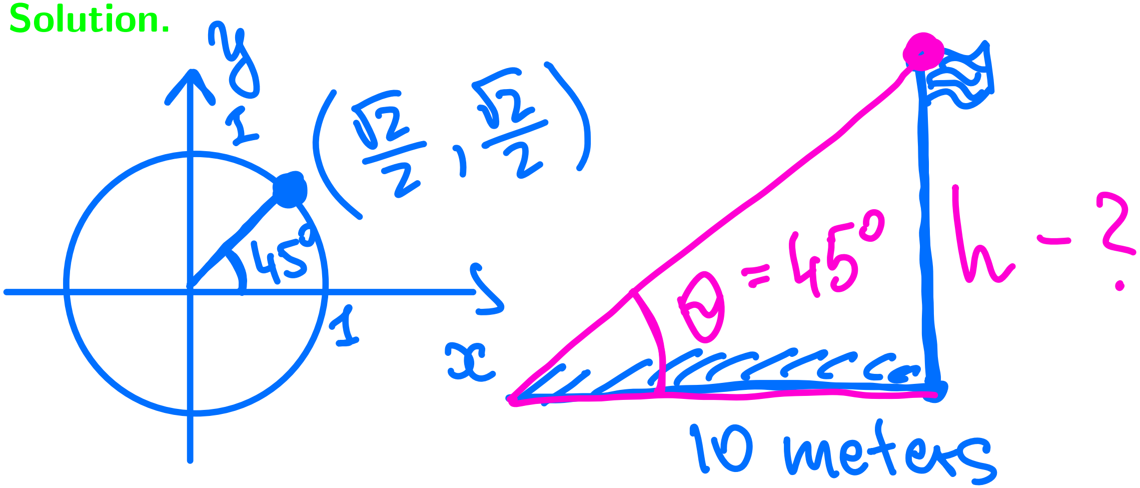
$$\tan(\theta) = \frac{Altitude}{Baseline}$$

Hence,

$$Altitude = \tan(\theta) \cdot Baseline$$

Example 1. A flagpole casts a shadow 10 meters long. The angle the shadow and the tip of the flagpole make with the ground is measured and is found to be 45°. What is the height of the flagpole?

Solution.



$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{\sqrt{2}/2}{\sqrt{2}/2}$$

$$\tan\theta = 1$$

$$h = \tan\theta \cdot 10 = \tan 45° \cdot 10 =$$

$$= 1 \cdot 10 = \boxed{10} \text{ meters.}$$

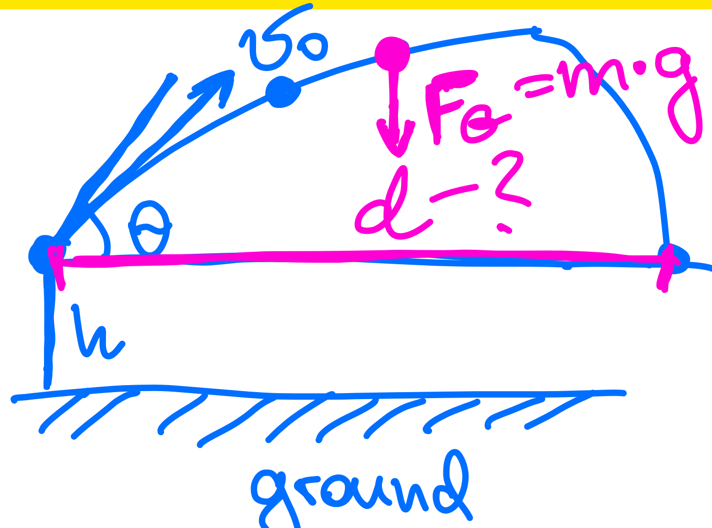
Remarks:

1. Rockets flown on windy days will usually not go straight up and will not go high as they could have gone.
2. To minimize errors in altitude measurements for rockets going into the wind, station the tracker at right angles to the wind flow.

Problem Description

- Let's try to write a program that simulates the flight of the projectile.
- We are interested in how far the projectile will travel when it starts its movement at various launch angles and initial velocities.

Program specification



The input to the program will be:

- the launch angle (in degrees);
- the initial velocity (in m/s);
- the initial height (in meters).

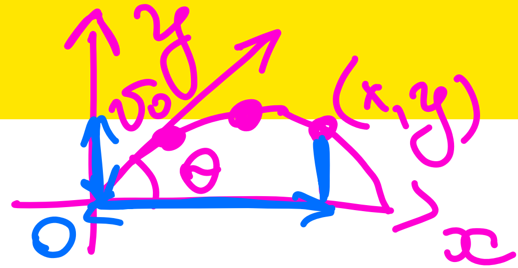
The output will be:

- the distance that the projectile travels before striking the ground (in meters).

The acceleration of gravity near the Earth's surface is roughly 9.8 m/s^2.

If an object is thrown straight up at 20 m/s, after one second it will be travelling upwards at 10.2 m/s. After another second, its speed will be 0.4 m/s. Shortly after that the object will start coming back down to Earth.

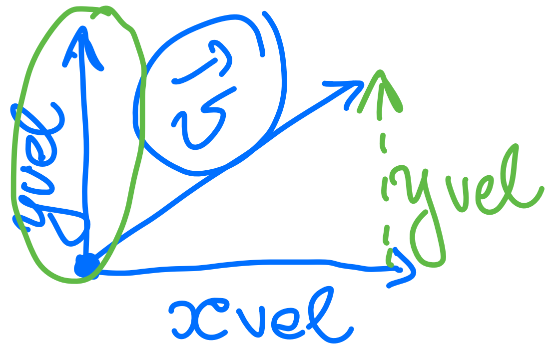
Program specification



$x(t)$
$y(t)$

- Given the nature of the problem, we need to consider the flight of the projectile in two dimensions: it's height and the distance it travels.
- We think of the position of the projectile as the point (x, y) where x is the distance from the starting point and y is the height above the ground.
- We suppose that the object starts at $(0, 0)$, and we want to check its position every tenth of a second.
- In that time interval it will have moved some distance upward ($y > 0$) and some distance forward ($x > 0$). The exact distance will be determined by the velocity in that direction.
- We are ignoring wind resistance.
- However, y will change over time due to gravity. The y velocity will start out positive and then become negative as the object starts to fall.

$$v = \frac{s}{t} \Rightarrow \boxed{s = v \cdot t}$$



$$\boxed{\vec{v} = xvel + yvel}$$

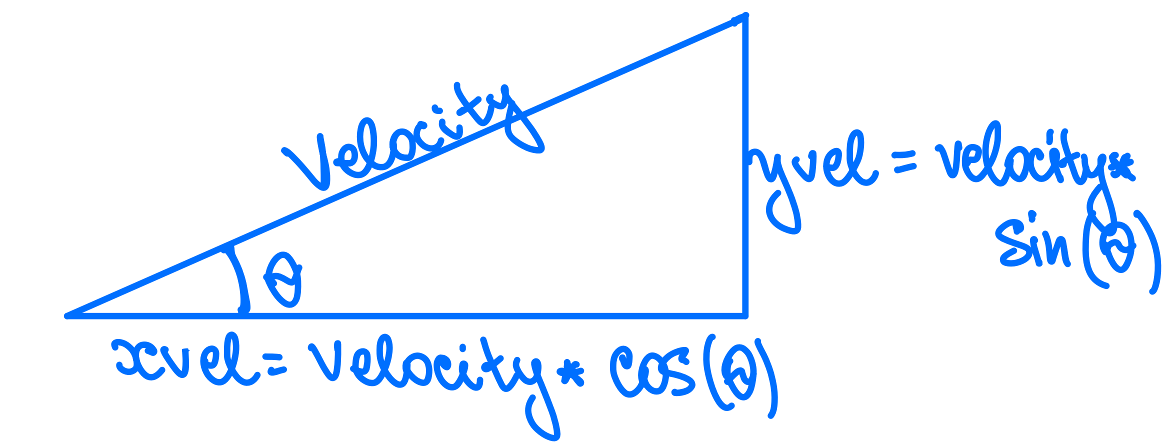
$$S \rightarrow xpos$$
$$S \rightarrow ypos$$

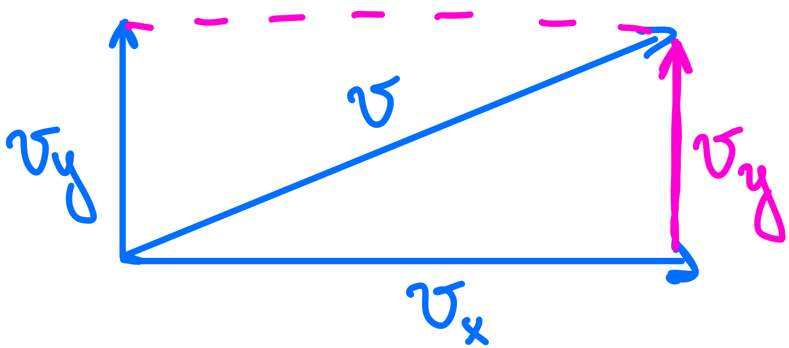
$$xpos = t \cdot xvel$$
$$ypos = t \cdot yvel$$

Program Algorithm

- Input: **angle, velocity, height, interval**.
- Calculate the initial position of an object: **xpos, ypos**.
- Calculate the initial velocities of an object: **xvel, yvel**.
- While the object is still flying: update **xpos, ypos** and **yvel** for interval seconds further into the flight.
- Output the distance traveled as **xpos**.

Geometry



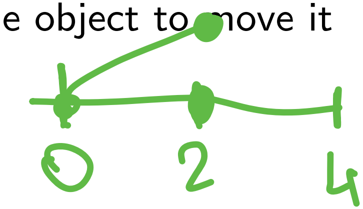


triangle law

$\vec{v} = \vec{v}_x + \vec{v}_y$

Particular details

- Each time through the loop we want to update the state of the object to move it time seconds farther.



- $$xpos = xpos + time * xvel$$
  .

xpos=0+2

- Each second, *yvel* must decrease by 9.8 m/s, the acceleration due to gravity.
- $$yvel = yvel - time * 9.8$$
  .

xpos=2+2=
=4

- To calculate how far the object travels over the interval, we need to calculate its *average* vertical velocity over the interval.
- Since the velocity due to gravity is constant, it is simply the average of the starting and ending velocities times the length of the interval:

$$ypos = ypos + time * (yvel + yvel1)/2.0$$

Python Code

```
def main():
angle = float(input("Enter the launch angle (in degrees):"))
vel = float(input("Enter the initial velocity (in m/s):"))
h0 = float(input("Enter the initial height (in meters):"))
time = float(input("Enter the time interval between position calculations:"

radians = (angle*pi)/180.0
xpos = 0
ypos = h0
xvel = vel*cos(radians)
yvel = vel*sin(radians)
while (ypos>=0.0):
xpos = xpos + time*xvel
yvel1 = yvel - 9.8*time
ypos = ypos + time*(yvel+yvel1)/2.0
yvel = yvel1
print("Distance traveled: 0:0.1f meters.".format (xpos))

main()
```

$\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$

$\sqrt{2}$

π

$e = 2.7$

real

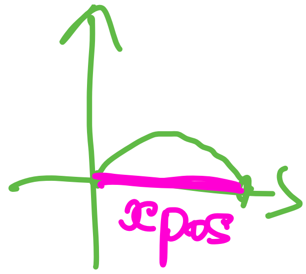
float

$\frac{30^\circ \cdot \pi}{180^\circ} = \frac{1}{6} \cdot \pi = \frac{\pi}{6} = \frac{3.14}{6} \approx$

Extra problems:

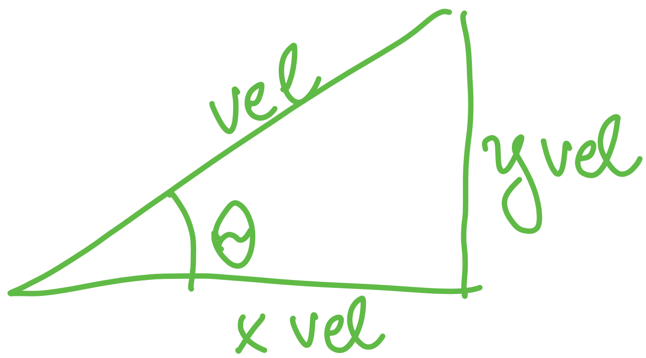
- calculate ypos for

$\theta = 90^\circ$
$v_0 = 30$
$t = 3$
$h_0 = 0$



- calculate time when the object will hit the ground

$$\boxed{ypos = 0}$$



$$xvel = vel \cdot \cos(\theta)$$
$$yvel = vel \cdot \sin(\theta)$$

while (. . .) do

repeat ...... until

for

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Thank you for your attention!